GCE AS/A level

0977/01

MATHEMATICS FP1

Further Pure Mathematics

A.M. THURSDAY, 14 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$S_n = \sum_{r=1}^{n} r(r^2 - 1),$$

obtain an expression for S_n in terms of n, giving your answer as a product of linear factors. [5]

2. The complex number z satisfies the equation

$$z(2 + \mathrm{i}) = (1 + 2\mathrm{i})^2.$$

(a) Express z in the form $x + \mathrm{i}y$. [6]

(b) Find the modulus and argument of z. [3]

3. The roots of the quadratic equation $2x^2 + x + 2 = 0$ are denoted by α, β.

(a) Show that

$$\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha} = \frac{11}{8}.$$ [5]

(b) Find the quadratic equation whose roots are $\frac{\alpha^2}{\beta}, \frac{\beta^2}{\alpha}$. [3]

4. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix}.$$

(a) (i) Find the adjugate matrix of **A**.
(ii) Find the inverse of **A**. [6]

(b) **Hence** solve the equations

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 1 & 4 \\ 4 & 5 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 7 \\ 10 \end{bmatrix}.$$ [2]

5. *(a)* Determine the value of k for which the following system of equations is consistent.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ k \end{bmatrix}$$ [5]

(b) Find the general solution for this value of k. [3]

6. Use mathematical induction to prove that $n^3 + 2n$ is divisible by 3 for all positive integers n. [7]

7. The transformation T in the plane consists of a reflection in the line $y = x$ followed by a translation in which the point (x, y) is transformed to the point $(x - 2, y + 2)$ followed by a reflection in the x-axis.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Find the coordinates of the fixed point of T. [4]

8. The function f is defined, for $x > 0$, by

$$f(x) = x^x.$$

(a) Use logarithmic differentiation to obtain an expression for $f'(x)$ in terms of x. [4]

(b) Determine the coordinates of the stationary point on the graph of f. [3]

(c) Show that

$$f''(x) = x^{x-1} + x^x(1 + \ln x)^2$$

and hence classify the stationary point as a maximum or a minimum. [4]

9. The complex numbers z and w are represented by points $P(x, y)$ and $Q(u, v)$ respectively in Argand diagrams and

$$wz = 1.$$

(a) Show that

$$x = \frac{u}{u^2 + v^2}$$

and obtain an expression for y in terms of u and v. [3]

(b) The point P moves along the line $y = mx + 1$.

(i) Show that the locus of Q is a circle.
(ii) Determine the radius and the coordinates of the centre C of the circle.
(iii) Write down the equation of the locus of C as m varies. [7]